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FREAK EDGE WAVES

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Abstract: The nonlinear and unsteady dynamics of the edge waves is discussed. Two physical processes: dispersive focusing and nonlinear modulational instability, are studied. Both mechanisms can induce the appearance of the short-living large-amplitude isolated waves and intense wave packets ("freak edge waves").

INTRODUCTION

Wave propagation in the inhomogeneous media can induce the scattering of the wave energy, as well as its capturing. The last phenomenon has a great interest due to the weak attenuation of the waves over long distances. There is a lot of tsunami observations when strong intensity can be explained with the theory of the trapped waves only. For instance, Ishi and Abe (1980) suggested that the manifestation of the catastrophic 1952 Kamchatka tsunami on the Japanese coast is related with the trapped waves. The 25 April 1992 Cape Mendocino earthquake generated a tsunami characterized by both coastal trapped edge wave and non-trapped tsunami modes that propagated north and south along the U.S. West Coast (Gonzales et al, 1995; Fujima et al, 2000). Totally, approximately 70% of the tsunami wave energy propagates along the Kurile Islands in Pacific as the trapped waves (Fine et al, 1983). Due to frequency dispersion of the trapped waves, such waves approach significantly later then the leading wave, and their amplitudes are significantly higher. Coastally trapped waves are an important component in the sea disturbances produced by cyclones moving along coastlines (Tang and Grimshaw, 1995). Short-scale edge waves may be generated from normally incident wind waves due to strong nonlinearity of the wind waves (Guza & Davis, 1974; Foda & Mei, 1981; Agnon & Mei, 1988).

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The mechanisms of the formation of the short-lived anomalous high edge waves ("freak" waves) are discussed in present paper. The first mechanism discussed early in papers (Aceev et al, 2001; Kurkin and Pelinovsky, 2003; Dubinina et al, 2003; Kurkin and Pelinovsky, 2004) is the dispersive focusing of the frequency modulated wave packets due to difference in the group velocities of propagated edge waves. This mechanism is effective for the edge waves on cylindrical bottom of any profile. The second mechanism is the modulational instability of the periodic wave train, which is known as the Benjamin-Feir instability for the Stokes waves. The analysis for edge waves above a beach of constant slope shows that the edge waves of any carrier frequency and modal number are unstable (Kurkin and Pelinovsky, 2004; Dubinina et al, 2004, 2005). For more complicated beach profiles the waves can be stable or unstable depending from the wave frequency. This mechanism leads to appearance of the group of anomalous waves in the almost periodic wave train for short time. All discussed mechanisms can induce an unusual sudden flooding on the coasts. Some observed data of such a flooding are discussed also.

DISPERSIVE FOCUSING OF EDGE WAVE TRAINS

Let us consider the wave motion above the cylindrical bottom in the framework of the linear shallow water theory

$$\frac{\partial^2 \eta}{\partial t^2} - g \cdot div(h \nabla \eta) = 0 , \qquad (1)$$

where $\eta(x,y,t)$ is the surface displacement, g is the gravity acceleration, $h(y)$ is the water depth, y is the offshore coordinate and x is the alongshore coordinate. The boundary conditions (on offshore coordinate) correspond to the wave vanishing on infinity and its bounding on the shoreline.

The wave equation (1) should be solved with the initial conditions. For tsunami problem the piston model is the popular model, thus the initial conditions correspond to the initial displacement of the sea surface

$$\eta(x,y,t=0) = \eta_0(x,y) , \qquad \frac{\partial \eta}{\partial t}(x,y,t=0) = 0 , \qquad (2)$$

but other initial conditions can be easily considered due to linearity of the wave equation. The solution of the Cauchy problem for the wave equation (1) can be expressed in the integral form. Physically, this solution is presented as a sum of the near field (algebraically attenuated from the source) and the wave component given the superposition of the edge waves. Far from the source, the wave component contributes mainly in the resulting field. We will consider the trapped waves far from the source and ignore the near field. The wave component of the general solution presents by the Fourier series,

$$\eta(x,y,t) = \sum_{n=0}^{n=\infty} \int_{-\infty}^{+\infty} A_n(k) F_n(k,y) \exp\left(i(\omega_n t - kx)\right) dk , \qquad (3)$$

where $A_n(k)$ can be easily found from $\eta_0(x,y)$ by inverse Fourier transformation. The function, $F_n(k,y)$ and the wave frequency, $\omega_n(k)$ are determined by the eigenvalue problem

$$\hat{\Gamma}F = \frac{d}{dy}\left[h(y)\frac{dF}{dy}\right] + \left(\frac{\omega^2}{g} - h(y)k^2\right)F = 0 \qquad (4)$$

with corresponding boundary conditions. For instance, for the beach of constant slope, $h(y) = \alpha y$ the offshore structure of the edge waves is described by

$$F_n(k,y) = \exp(-ky)L_n(2ky), \qquad \omega_n = \sqrt{(2n+1)\alpha g k}, \qquad (5)$$

where $L_n(ky)$ is the Laguerre polynomial. The offshore structure and dispersion relation of the Stokes edge waves are shown in Fig. 1 for $\alpha = 10^{-3}$.

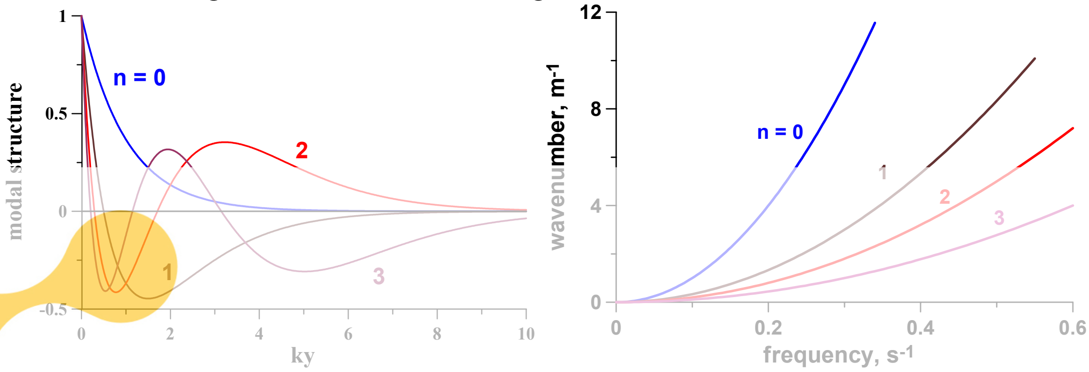


Fig. 1. Offshore structure and dispersion relation for Stokes edge waves

Thus, the linear superposition of the free edge waves (3) is the mathematical model for discussion of the anomalous high wave appearance. It is important to note that the edge waves have in general case the significant dispersion; in particular, for the Stokes waves dispersion relation (5) coincides with the dispersion relation for the wind waves in deep water (with reduced value of the gravity acceleration). Dispersion is usually considered as the mechanism of the wave attenuation due to transformation of the initial impulse into the wave train with decreasing amplitude and increasing length. It is evident that if at the initial moment the wave packet has the slow waves in its front and the fast waves on its end, the fast waves will overtake the slow waves. In the moment of overtaking (wave focus) the individual waves merge with forming of the large amplitude pulse. So, the focusing process is the inverse process of the dispersive attenuation. Mathematically, it follows from the invariance of the wave equation (1) on the sign changing of time and coordinates (in fact, only the propagated coordinate, alongshore coordinate should be considered). Therefore, the complicated mathematical problem to prove the appearance of the anomalous pulse from the given wave field can be reduced to the simpler Cauchy problem of the evolution of the anomalous pulse. All obtained solutions after inverting in space will present the wave packets evolved into the anomalous high impulse.

The main idea of analysis of appearance of the freak waves is to use the initial condition for (3) in the form of possible large pulse. Such a wave is described by the Fourier integral

$$\eta_{fr}(x,y) = \eta(x,y,0) = \sum_n \int A_n(k)F_n(k,y)\exp(ikx)dk, \qquad (6)$$

where $A_n(k)$ is a spectrum of anomalous high (freak) wave. The wave field on large times can be found using the stationary phase method

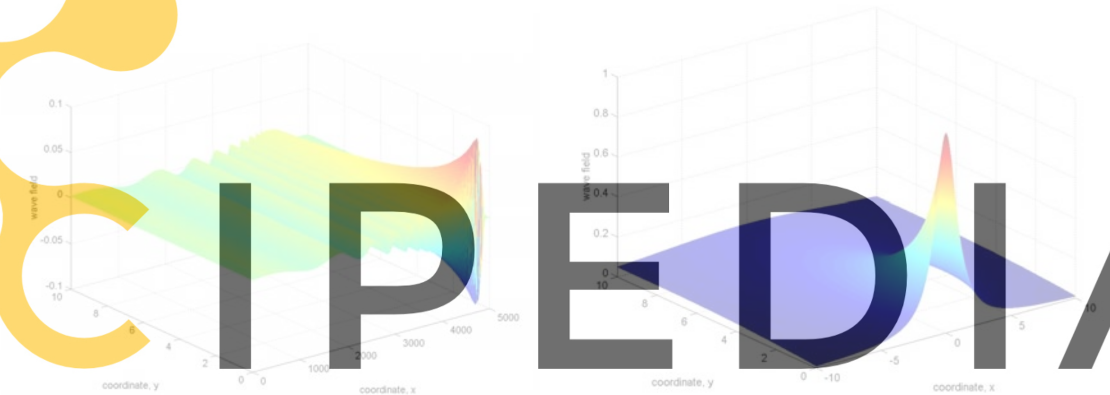
$$\eta(x,y,t) \approx \sum_n \frac{A_n(k)F_n(k,y)}{\sqrt{2\pi t\,|\,dc_n/dk\,|}} \exp\big(i(\omega_n t - kx - \pi/4)\big), \qquad (7)$$

where c_n is the group velocity determined by

$$c_n = \frac{d\omega_n}{dk} = \frac{x}{t}. \qquad (8)$$

For the fixed time, the wave number as a function of coordinate, *k(x)* is found from (8), and then the wave field from (7), it is function of both spatial coordinates.

Inverting time, *t* and coordinate, *x* in (7), the wave packet will have the slow waves on its front and fast waves in back. Such wave packet will evolve with time in the localized large pulse (6), and then disperse in the wave packet (7). Fast long waves will be again in front of more slow short waves. Fig. 2 illustrates the process of the appearance and disappearance of the freak wave. It is evident, that the significant amplification is in the vicinity of the wave focusing only; so, the anomalous high wave is rapidly appeared and also rapidly disappeared, and it has short-lived time (Fig. 3).



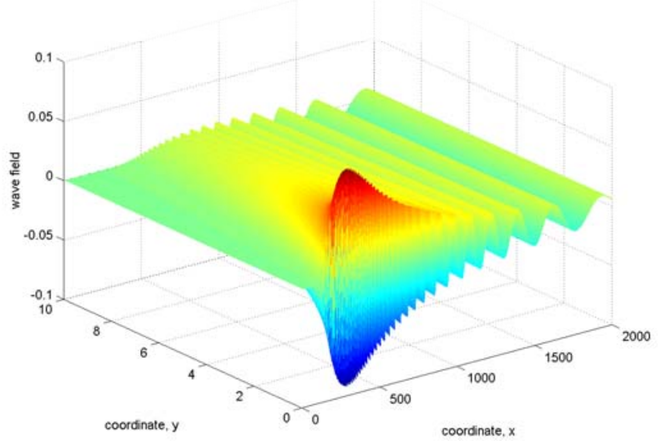


Fig. 2. Appearance and disappearance of the freak wave

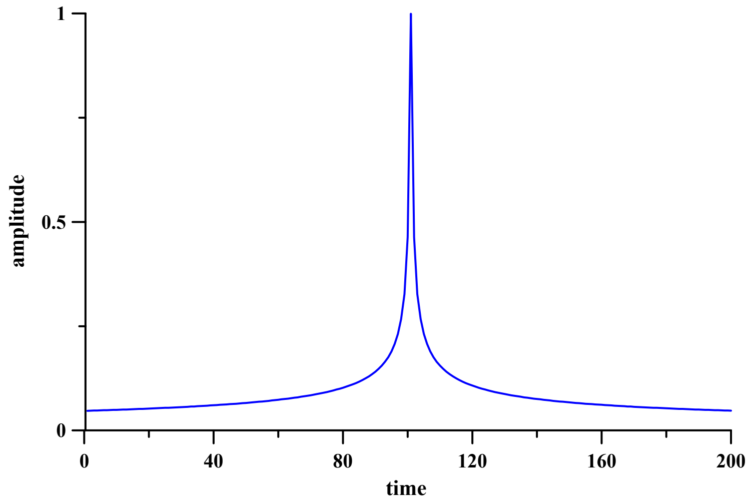


Fig. 3. Maximal value of the amplitude of the wave train versus time

Deterministic edge waves are considered above. In the framework of the linear theory the random and regular components propagate independently. As a result, the freak edge wave may be formed on the background of the random sea, as it was shown early for the Stokes edge waves (Kurkin and Pelinovsky, 2002).

NONLINEAR SELF-MODULATION OF EDGE WAVE TRAINS

Let us consider now nonlinear equations of the shallow-water theory

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left((h(y)+\eta)\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left((h(y)+\eta)\frac{\partial \Phi}{\partial y}\right) = 0, \quad (9)$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \Phi}{\partial y}\right)^2 + g\eta = 0, \quad (10)$$

where $\Phi(x, y, t)$ is the potential determined the alongshore current, $u = \partial\Phi/\partial x$, and offshore current, $v = \partial\Phi/\partial y$. Nonlinear modulational instability of the lowest mode of the Stokes edge waves was investigated in several papers (Whitham, 1976; Minzoni and Whitham, 1977; Yeh, 1985) where nonlinear dispersion relation was derived. Here this analysis is extended for the edge waves of any modal number.

The solution of the equations (9) and (10) is seeking in the form of the progressive steady waves considering η and Φ as functions of $\theta = kx - \Omega t$ and y. Assuming the smallness of the wave amplitude, a, the wave field is expanded by the asymptotic series

$$\eta = a\{\eta_1(\theta, y) + ka\eta_2(\theta, y) + k^2a^2\eta_3(\theta, y) + \ldots\}, \quad (11)$$

$$\Phi = a\Omega^{-1}\{\Phi_1(\theta, y) + ka\Phi_2(\theta, y) + k^2a^2\Phi_3(\theta, y) + \ldots\}. \quad (12)$$

The wave frequency, Ω should be presented also as power series in the amplitude

$$\Omega^2 = \omega_n^2\{1 + \gamma_n k^2 a^2 + \ldots\}. \quad (13)$$

We will not give here the complicated technical details of the calculations of the nonlinear dispersion relation in the third order of the perturbation theory; see for instance, Dubinina et al (2004). In particular, for the beach of constant slope, the coefficient γ_n depends from modal number, n only, and can be approximated by the regression curve

$$\gamma_n = \frac{1}{2+8n}, \tag{14}$$

its accuracy is demonstrated in Fig. 4. In particular, for n = 0 this expression was derived by Whitham, 1976. It is important to mention that the nonlinear coefficient has the same sign for all values of the modal number, and, therefore, all modes of the Stokes edge waves are unstable.

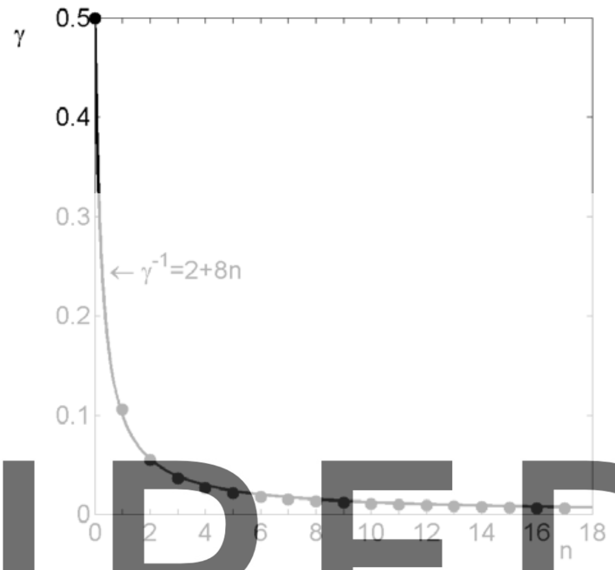


Fig. 4. Coefficient γ versus modal number

Based on the nonlinear dispersion relation, all coefficients of the nonlinear Schrodinger equation for the complex wave amplitude are calculated in an explicit form, so that this nonlinear evolution equation can be specified as

$$i\left[A_\tau + \frac{1}{2}A_\xi\right] - \frac{1}{8}A_{\xi\xi} - \frac{1}{2}\gamma_n |A^2| A = 0, \tag{15}$$

where $A = ka$, $\xi = kx$, and $\tau = \omega t$; k and ω are wave number and frequency of the carrier wave. Early this equation has been derived for the lowest mode of the Stokes wave only (Akylas, 1983).

Fig. 5 demonstrates the developing of the modulational instability for the initial weak disturbance of the wave amplitude

$$A(\xi, \tau = 0) = A_0(1 + m\cos(K\xi)), \tag{16}$$

where m is the coefficient of modulation, K is the wave number of modulation taken from the condition of modulational instability. In the case of the weak amplitude wave train (Fig. 5, left), the one intense wave packet is appeared and disappeared on the background of the almost periodic wave. Increasing of the wave amplitude leads to the appearance of several intense wave groups (Fig. 5, right).

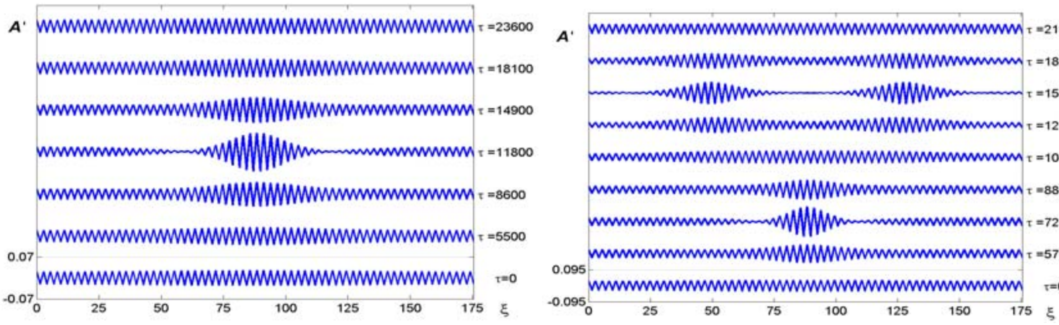


Fig. 5. Developing of the modulational instability

For comparison, the process of the dispersive focusing is also modelled in the framework of the nonlinear Schrodinger equation (15); in this case the initial disturbance is

$$A = A_0 \exp(-(\xi - \xi_0)^2 / d^2)\,, \tag{17}$$

for $A_0 = 0.07$ and $d = 5$. Fig. 6 displays the wave focusing of the frequency modulated wave train. As it can be seen, both mechanisms: modulational instability and nonlinear-dispersive focusing, result to appearance of the freak edge waves.

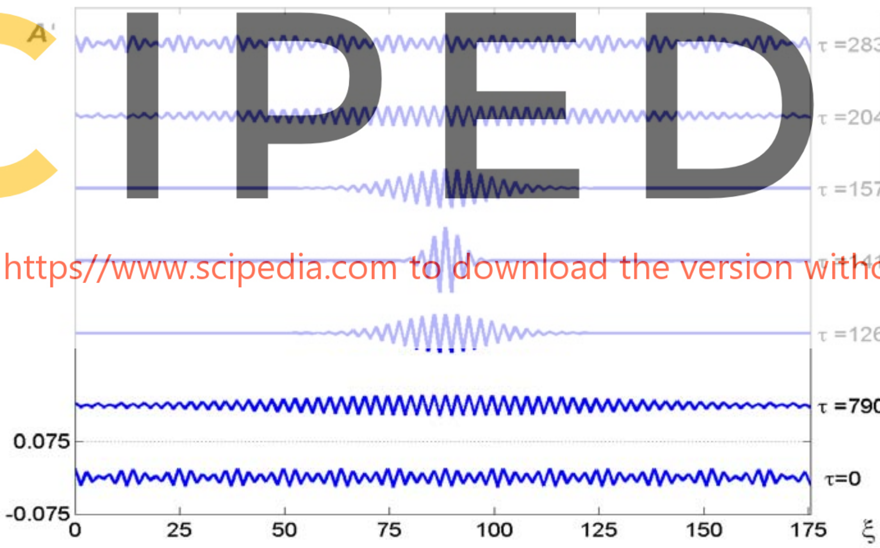


Fig. 6. Nonlinear – dispersive focusing of the wave train

Similarly (Lechuga, 1996) studied Benjamin-Feir instability and the resulting modulation from another point of view in a more restricted case , mainly standing or quasi standing edge waves of zero mode on a planar beach. In this paper the approach was constructive using Hill equation to check domains of instability.

CONCLUSIONS

Nonlinear and unsteady dynamics of the edge waves can induce the appearance of the short-living large-amplitude localized pulses (freak edge waves). They can appear as a result of the action of two physical mechanisms: dispersive focusing and nonlinear modulational instability. Both processes are studied and their effectiveness is analyzed. How to anticipate the flooding of the coastal zone as resulting of these mechanisms of generating freak edge waves should be the practical conclusion of this paper.

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